MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

ECT1026 – FIELD THEORY

(TE/RE)

14 OCTOBER 2017
9.00 a.m - 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 8 pages with 4 Questions only.

2. Attempt **ALL (4)** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

(a)(i) Define a unit vector of a vector $\vec{B}$. [2 marks]

(ii) Explain the situation of vectors $\vec{A}$ and $\vec{B}$ when the dot product of two vectors is zero, i.e. $\vec{A} \cdot \vec{B} = 0$. [2 marks]

(iii) Determine the unit vector $\hat{n}$ that points along the direction of $\vec{A} \times \vec{B}$. Given the vectors $\vec{A} = -\hat{x} - 2\hat{y} + 4\hat{z}$ and $\vec{B} = -\hat{x}+3\hat{y} - 2\hat{z}$. [5 marks]

(b) The vector field strength is measured by the number of flux lines passing through a unit surface normal to the vector. The flux of a vector field is analogous to the flow of an incompressible fluid such as water.

(i) Define the divergence of a vector field $\vec{B}$ at a point. [2 marks]

(ii) What are the physical meanings of net positive and negative divergences? [2 marks]

(iii) Given that $\vec{E} = r\hat{r} + sin\theta\hat{\theta}$, find the expression of $\vec{\nabla} \cdot \vec{E}$ in spherical coordinate system. [5 marks]

(c)(i) State the Stokes's theorem. [2 marks]

(ii) Evaluate the $\int_S (\vec{\nabla} \times \vec{D}) \cdot d\vec{s}$ over the surface of the semi-circle with radius of $r = 3$. Assuming the vector field $\vec{D} = rcos\phi\hat{r} + sin\phi\hat{\phi}$. [5 marks]

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Question 2

(a)(i) Explain the volume charge density ρ_v in a Cartesian coordinate system. [2 marks]

(ii) Give an expression for total charge Q contained in a volume v in Cartesian coordinate system. [2 marks]

(iii) Consider the bulk piece of germanium with cross section of 1 cm^2 in Figure Q2. Assume that the electron charge density is $\rho_e = -4 \times 10^{-6} C/cm^3$ and moves with velocity $\vec{v}_e = -15\hat{x}$cm/s. The hole charge density is $\rho_h = +4 \times 10^{-6} C/cm^3$ and has velocity $\vec{v}_h = +10\hat{x}$cm/s. Determine the current through the semiconductor cross section. [4 marks]

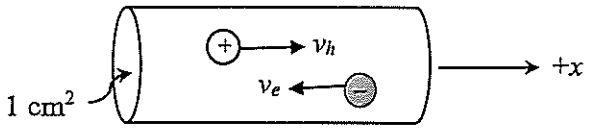

Figure Q2

(b)(i) Define the electric field at any point using the electric force. [2 marks]

(ii) State the dependence of electric force. [2 marks]

(iii) An unknown source distribution exerts a force $\vec{F} = -8\hat{y} + 3\hat{z}$ pN on a test charge $Q = 2$ pC that is at rest at a point. Find an expression of electric field at that point. [4 marks]

(c)(i) Briefly describe the Gauss' law for electric field computation. [2 marks]

(ii) Using the Gauss' law, calculate the electric field generated by a point charge placed in an infinite, homogenous region with permittivity ε. [3 marks]

(d) One of the most common transmission lines is the coaxial line consisting of inner and outer conductors. RG-58U coaxial cable has a 20-AWG solid inner conductor with a 0.406 mm radius, surrounded by a solid polyethelene dielectric and an outer, braided conductor of radius 1.553 mm. Calculate the capacitance per meter for this coaxial cable. Assume the dielectric constant of polyethlylene is $\varepsilon_r = 2.26$. [4 marks]

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Question 3

(a) State Biot-Savart Law. [2 marks]

(b) Figure Q3(a) shows a wire consisting of two straight sections of length L = 5 cm, a circular arc of radius R = 4 cm and central angle $\theta = \pi/3$. The wire carries a steady current I = 2 A.
Using Biot-Savart's law, find

(i) the magnitude of magnetic flux density $\vec{B}$ at point P due to the two straight sections of the wire only. Explain your answer. [3 marks]

(ii) the total magnetic flux density $\vec{B}$ at point P produced by the whole wire. [7 marks]

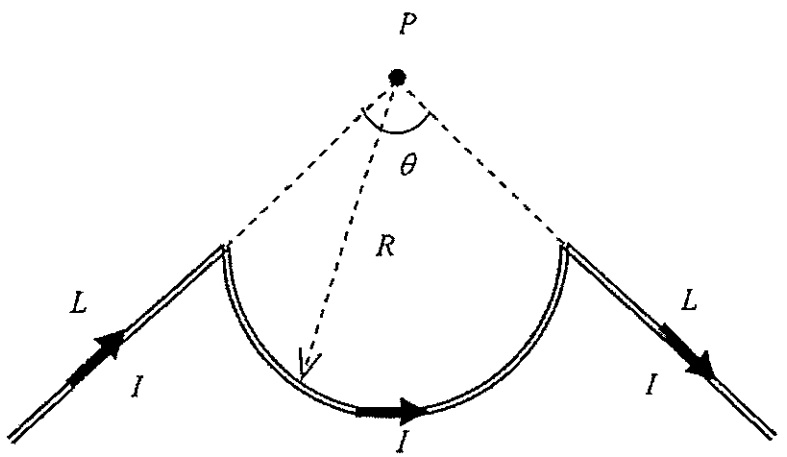

Figure Q3(a)

Continued .....

Question 3

(c) Figure Q3(b) shows a square loop of N close-packed turns positioned near a long straight wire carrying steady current I.

(i) Show that the mutual inductance M of the system is given by

$$M = \frac{N\mu_0 b}{2\pi} \ln\left(1 + \frac{b}{a}\right)$$ [10 marks]

(ii) Calculate the mutual inductance if a = 2 cm, b = 3 cm and the loop consists of 200 turns. [3 marks]

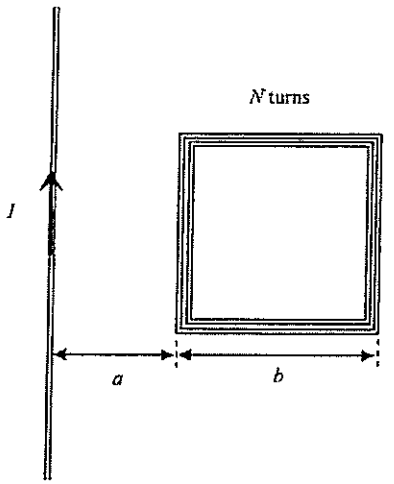

Figure Q3(b)

Continued .....

Question 4

(a) Sketch the hysteresis loop of a ferromagnetic material and label the following points.
(i) Saturation
(ii) Retentivity
(iii) Coercivity [4 marks]

(b) Explain why ferromagnetic material is always preferred as compared to paramagnetic and diamagnetic materials in magnetic systems. [3 marks]

(c) The magnetic circuit in Figure Q4(a) is made of **cast-iron** and has symmetrical left and right arms.
The dimensions of the core limbs are as follows:
$l_{ab} = l_{bc} = l_{cd} = 8$ cm
$l_{gap} = 0.5$ cm
$l_{ek} = 6$ cm
cross-sectional area = 2 cm × 2 cm

(i) Draw the equivalent magnetic circuit. [4 marks]

(ii) Using the B-H curve in Figure Q4(b), find the current I needed to establish flux Φ_2 of 60 μWb at the right arm of the core. [14 marks]

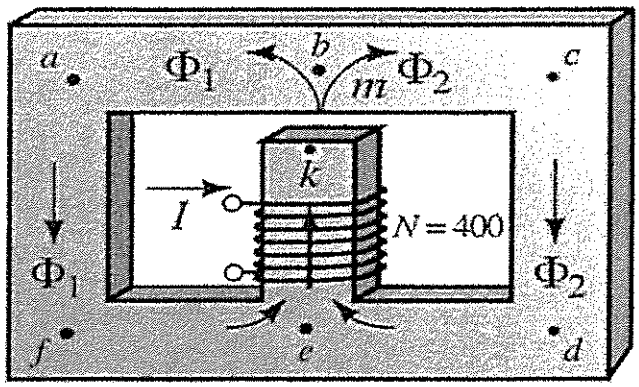

Figure Q4(a)

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Question 4

(c)

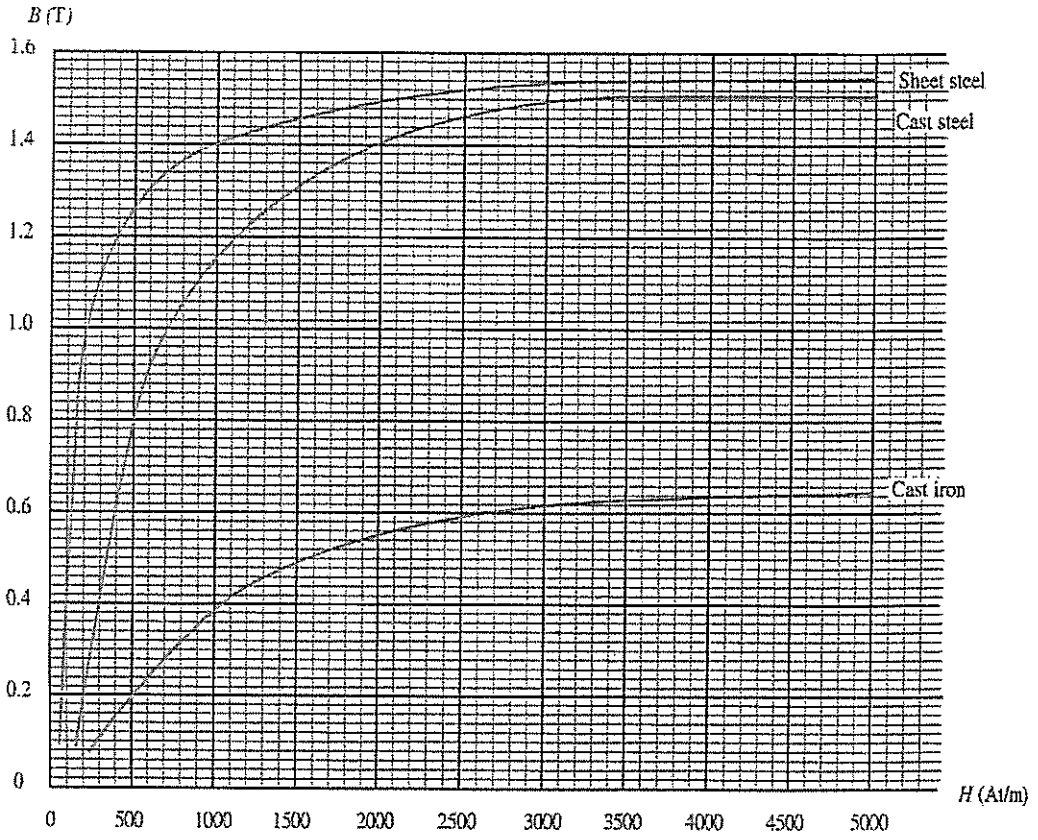

Figure Q4(b)

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Physical Constants, Vector and Coordinate Transformation Relations

Elementary charge	e	1.60×10^{-19} C
Permittivity of free space	ε_0	8.85×10^{-12} F/m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\bar{s}_x = \hat{x}dydz$ $d\bar{s}_y = \hat{y}dxdz$ $d\bar{s}_z = \hat{z}dxdy$	$d\bar{s}_r = \hat{r}rd\phi dz$ $d\bar{s}_\phi = \hat{\phi}drdz$ $d\bar{s}_z = \hat{z}rdrd\phi$	$d\bar{s}_R = \hat{R}R^2\sin\theta d\theta d\phi$ $d\bar{s}_\theta = \hat{\theta}R\sin\theta dRd\phi$ $d\bar{s}_\phi = \hat{\phi}RdRd\theta$
Differential volume	$dxdydz$	$rdrd\phi dz$	$R^2\sin\theta dRd\theta d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to Spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to Cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$